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SLOW MOTIONS OF A SOLID IN A CONTINUOUSLY STRATIFIED FLUID

V. A. Vladimirov and K. I. Il'in

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The problem of the motion of a solid in a fluid of nonuniform density (stratified) is a particularly difficult and intensively investigated one [1]. The almost total absence of exact solutions has led to a great deal of work on the development of linear-approximation models [2-5].

Again in the linear approximation we have constructed particular solutions of the flow problem for the slow motions of a three-dimensional or two-dimensional solid in an ideal incompressible stratified fluid. The shape of the body and its direction of motion may be arbitrary, and the dependence of the velocity on time t has a special form [proportional to $\exp(\alpha t)$ with constant $\alpha > 0$]. A new method of constructing the solution is proposed. It is based on the following remarkable fact: by direct transformation the problem can be reduced to the classical problem of the potential flow of a homogeneous fluid past some other fictitious body. This equivalence makes it possible to calculate the velocity and resistance fields in the stratified fluid. And the formulas for the resistance are simple analytic expressions.

The limiting solutions as $\alpha \to 0$, which are of interest from two points of view, have been studied in detail. First, they correspond to the important practical case of uniform motion, and, second, they coincide with the solutions of the problem of the instantaneous setting in motion of a body initially at rest. At the same time, the problem of impulsive motion, previously considered in various particular formulations [3, 5], has been solved in general form. The calculations showed that the limiting ($\alpha \to 0$) flows have a characteristic layered structure. The vertical velocity component is equal to zero, and the fluid moves in horizontal layers (z = const). In all cases the resistance to the uniform motion of a three-dimensional body (less the buoyancy force) is equal to zero, which gives a result analogous to the D'Alembert paradox. For a two-dimensional body a fundamentally different answer is obtained: in the limit as $\alpha \to 0$ the resistance is finite for both horizontal and vertical motion.

Thus, a number of general results relating to low Froude number regimes have been obtained for stratified flow past a body. The analogous problem of the motion of a body in a rotating fluid was solved in [6]. In the light of the analogy between stratification and rotation [7, 8] our results are a development of the approach adopted in [6].

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1. Formulation of the Problem. We will consider a three-dimensional solid moving in an unbounded ideal incompressible fluid of nonuniform density. The equations of motion of the fluid are taken in the Boussinesq approximation [9]:

$$D\mathbf{u} = -\nabla p + \rho \mathbf{g},\ D\rho = 0,\ \operatorname{div} \mathbf{u} = 0,\ D \equiv \partial/\partial t + \mathbf{u}\cdot\nabla \tag{1.1}$$

($\mathbf{u}$, ρ, and p are the velocity, density, and pressure fields, $\mathbf{g}$ is the uniform gravity field). In the Cartesian coordinate system x, y, z we take $\mathbf{u} = (u, v, w)$, $\mathbf{g} = (0, 0, -g)$, $g > 0$. At $r \equiv \sqrt{x^2 + y^2 + z^2} \to \infty$ the fluid is at rest and in a state of stable hydrostatic equilibrium with density profile $\rho_0(z)$:

$$\mathbf{u} \to 0,\ \rho \to \rho_0(z) \text{ as } r \to \infty. \tag{1.2}$$

On the boundary of the body the no-flow conditions

$$(\mathbf{u} - \mathbf{u}_*)\nabla f = 0, \tag{1.3}$$

are satisfied, where $\mathbf{u}_*(t)$ is the velocity of the body; in the coordinate system moving with the body the boundary $\partial\tau$ is given by the equation

$$f(x, y, z) = 0. \tag{1.4}$$

We will study the exponential regimes

$$\mathbf{u}_*(t) = \varepsilon \mathbf{u}_0 \exp(\alpha t),\ \alpha > 0 \tag{1.5}$$

($\mathbf{u}_0 = (u_0, v_0, w_0)$, $|\mathbf{u}_0| = 1$, α and ε are constants). Moreover, it is assumed that as $t \to -\infty$ the fluid (like the body) is at rest:

$$\mathbf{u}(r, t) \to 0,\ t \to -\infty. \tag{1.6}$$

It is required to determine the motion of the fluid and the force acting on the body when the velocity of the body $|\mathbf{u}_*|$ is small.

In the coordinate system moving with the body equations (1.1) take the form:

$$D\mathbf{u} = -\nabla p - \alpha \mathbf{u}_* + \rho \mathbf{g},\ D\rho = 0,\ \operatorname{div} \mathbf{u} = 0, \tag{1.7}$$

and the boundary conditions (1.2), (1.3) give

$$\begin{gathered} \mathbf{u}\nabla f = 0 \text{ when } f = 0, \\ \mathbf{u} \to -\mathbf{u}_*(t),\ \rho \to \rho_0(z + h(t)),\ r \to \infty,\ h(t) \equiv w_*(t)/\alpha. \end{gathered} \tag{1.8}$$

2. Slow Motion. The next basic assumption is that the motion of the body (1.5) is so slow that the equations of motion (1.7) can be taken in linearized form. This approximation corresponds to low Froude numbers (Fr $\ll$ 1) and high Reynolds numbers (Re $\gg$ 1). Since these two conditions are not inconsistent, the regimes in question may exist in reality.

The linearization of Eqs. (1.7) gives

$$\begin{gathered} u_t = -p_x - \alpha u_*,\ v_t = -p_y - \alpha v_*, \\ w_t = -p_z - \rho g - \alpha w_*,\ \rho_t + \rho_0'(w + w_*) = 0,\ u_x + v_y + w_z = 0, \end{gathered} \tag{2.1}$$

where u, v, and w are the velocity components in the coordinate system moving with the body, $\mathbf{u}_* \equiv (u_*, v_*, w_*)$, the pressure and density perturbation fields are determined as corrections to the basic state $p_0(z + h(t))$, $\rho_0(z + h(t))$, and a prime denotes the derivative with respect to the variable z. Of course, there always exist values of t so large that for the law of motion (1.5) linearization will be incorrect. Therefore the results obtained will have physical significance only for times less than some value t_0.

The stratification is assumed to be linear $\rho_0' = \text{const}$. In this case the coefficients in Eqs. (2.1) will be constants, the time dependence remaining only on their right sides. The solution of the problem (1.6), (1.8), (2.1) will be found in the form:

$$(\mathbf{u}, p, \rho) = \varepsilon(\mathbf{u}_1, \alpha p_1, \rho_1)\exp(\alpha t) \tag{2.2}$$

with functions $\mathbf{u}_1$, p_1, and ρ_1 that depend only on $\mathbf{r} \equiv (x, y, z)$. Substituting (2.2) in (2.1) and eliminating ρ gives

$$\begin{gathered} u_1 + u_0 = -p_{1x},\ v_1 + v_0 = -p_{1y}, \\ (1 + N^2/\alpha^2)(w_1 + w_0) = -p_{1z},\ u_{1x} + v_{1y} + w_{1z} = 0 \end{gathered} \tag{2.3}$$

($N^2 \equiv -g\rho_0'$). The boundary conditions for (2.3) follow from (1.8):

$$\mathbf{u}_1 \to -\mathbf{u}_0 \text{ as } r \to \infty,\ \mathbf{u}_1\nabla f = 0 \text{ when } f = 0. \tag{2.4}$$

After appropriate stretching of the variables in (2.3) we can see that the pressure plays the role of velocity potential $\mathbf{u}_1 + \mathbf{u}_0$. Introducing the notation $\alpha \equiv kN$, $\varphi \equiv -p_1$, $\varkappa^2 \equiv (1+k^2)/k^2$, from (2.3) we obtain

$$\varkappa^2(\varphi_{xx} + \varphi_{yy}) + \varphi_{zz} = 0. \tag{2.5}$$

The boundary condition (2.4) takes the form:

$$\begin{gathered} \nabla\varphi \to 0 \text{ as } r \to \infty, \\ \varkappa^2(\varphi_x f_x + \varphi_y f_y) + \varphi_z f_z = \varkappa^2 \mathbf{u}_0 \nabla f \text{ when } f = 0. \end{gathered} \tag{2.6}$$

By means of the change of independent variables

$$\xi = x/\varkappa,\ \eta = y/\varkappa,\ \boldsymbol{\xi} \equiv (\xi, \eta, z) \tag{2.7}$$

we reduce the problem (2.5), (2.6) to the following form:

$$\begin{gathered} \varphi_{\xi\xi} + \varphi_{\eta\eta} + \varphi_{zz} = 0,\ \nabla_{\xi}\varphi \to 0 \text{ as } |\boldsymbol{\xi}| \to \infty, \\ \nabla_{\xi}\varphi\nabla_{\xi}f = \mathbf{U}\nabla_{\xi}f \text{ when } f = 0, \\ \nabla_{\xi} = (\partial/\partial\xi, \partial/\partial\eta, \partial/\partial z),\ \mathbf{U} = (\varkappa u_0, \varkappa v_0, \varkappa^2 w_0). \end{gathered} \tag{2.8}$$

It is a remarkable fact that (2.8) coincides with the problem of the motion of a solid in a homogeneous fluid with a potential flow regime. The function φ plays the part of velocity potential, $\mathbf{U}$ represents the velocity of the body, and the part of the new boundary of the body (1.4) is played by the surface $\partial\sigma$, given by the equation

$$f(\varkappa\xi, \varkappa\eta, z) = 0. \tag{2.9}$$

Thus, by stretching the variables (2.7) we have reduced the problem of the motion of a body in a stratified fluid to the fictitious motion of a body in a homogeenous fluid.

3. <u>Resistance.</u> The solution of (2.8) is usually represented in the form [7]:

$$\varphi = \Phi_i(\boldsymbol{\xi})U_i \tag{3.1}$$

(summation is carried out over the recurring vector and tensor indices). From (2.2), (3.1), and $\varphi \equiv -p_1$ we calculate the resistance

$$F_i = -\oint_{\partial\tau} p\, dS_i,\ F_i^* \equiv (F_1, F_2, F_3/\varkappa) = -\varepsilon k\varkappa N e^{kNt} M_{in} U_n, \tag{3.2}$$

where M_{in} is the apparent mass tensor in problem (2.8) which, as is known, is symmetric:

$$M_{in} = M_{ni} = -\oint_{\partial\sigma} \Phi_n\, dS_i. \tag{3.3}$$

In (3.3) the integration is carried out over the surface (2.9). In (3.2) the factors containing $\varkappa$ are obtained on going over from integration with respect to $\partial\tau$ to integration with respect to $\partial\sigma$. In the case of a triaxial ellipsoid with semiaxes a, b, c Eq. (2.9) can be written as

$$\varkappa^2\xi^2/a^2 + \varkappa^2\eta^2/b^2 + z^2/c^2 - 1 = 0.$$

The tensor M_{in} is diagonal with components given, for example, in [10]. From (3.2) there follows

$$\begin{gathered} F_1 = -\varepsilon k^{-1}(1+k^2)N e^{kNt} M_{11} u_0, \\ F_2 = -\varepsilon k^{-1}(1+k^2)N e^{kNt} M_{22} v_0,\ F_3 = -\varepsilon k^{-3}(1+k^2)^2 N e^{kNt} M_{33} w_0. \end{gathered} \tag{3.4}$$

It should be noted that although the calculation of the resistance has been reduced to the calculation of the fictitious apparent mass tensor (3.3), the work done by the force applied to the body (3.2), (3.4) goes towards increasing both the kinetic and the potential energy of the fluid. The presence of the tensor M_{in} in expression (3.2) merely reflects the fact that for the chosen law of motion (1.5) both forms of energy are described by means of this tensor. The energy integral for Eqs. (2.1), written in the initial coordinate system (fluid at infinity at rest), has the form

$$2E = \int \left(u^2 + v^2 + w^2 + \left(N^2/\rho_0'^2\right)\rho^2\right) dx\, dy\, dz.$$

From the law of motion (2.2) there follows $\alpha\rho = -\rho_0' w$; therefore $2E = \int (u^2 + v^2 + \varkappa^2 w^2)\, dx\, dy\, dz.$ Going over to the notation of problem (2.8), we obtain

$$2E = \varepsilon^2 e^{2\alpha t} M_{in} U_i U_n. \tag{3.5}$$

Thus, the kinetic and potential energies can be written in the form of a fictitious kinetic energy.

All the foregoing can be applied to the case of the motion of a two-dimensional body. For the resistance (per unit length along the y axis) instead of (3.2) we find

$$F_1 = -\varepsilon N k e^{kNt} M_{1n} U_n,\ F_3 = -\varepsilon N \sqrt{1+k^2} e^{kNt} M_{3n} U_n. \tag{3.6}$$

If the body is an elliptic cylinder with semiaxes a and c, then (3.6) takes the form:

$$(F_1, F_3) = -\pi \varepsilon N \sqrt{1+k^2} e^{kNt} (c^2 u_0, a^2 w_0). \tag{3.7}$$

4. Flow Structure for Uniform Motion. Motion at constant velocity, to which in (1.5) and the subsequent relations there corresponds the passage to the limit $\alpha \to 0$, $k \to 0$, is of considerable interest.

The velocity fields characteristic of this regime can be obtained by passing to the limit $k \to 0$ in the solutions of problem (2.8). This is easy to do when the explicit form of the solution for finite k is known (as, for example, for an ellipsoid or an elliptic cylinder). In general, the passage to the limit $k \to 0$ in the solutions of problem (2.8) can be made only for the vertical motions of a three-dimensional or two-dimensional body. This is possible due to the fact that for small k the deformed surface (2.9) for a three-dimensional body is a vertical "needle" with transverse dimension of the order of k, and for a plane body a vertical plate with thickness of the order of k. For small k this enables the problem (2.8) to be solved within the framework of the slender body approximation, which has been well worked out, and permits the passage to the limit $k \to 0$ in the solution obtained.

We will consider the motion of a three-dimensional body in the vertical direction. For simplicity, let the surface of the body possess axial symmetry and be described by the equation $r = f(z)$, $|z| < a$ $(r \equiv \sqrt{x^2+y^2})$. Moreover, let $\Phi = \varphi - Wz$, $W = \varkappa^2 w_0$, $\zeta = \sqrt{\xi^2+\eta^2}$. Then for the velocity potential Φ we obtain the well-known [11] problem of uniform flow past an axisymmetric slender body $\zeta = \varkappa^{-1} f(z)$ $(|z| < a)$ with velocity at infinity $\mathbf{U} = (0, 0, -W)$. Using the slender body approximation [11], for the radial σ and axial w velocity components we obtain the expressions

$$\sigma = -\frac{w_0}{2\varkappa} \int_{-a}^{a} f(z') f'(z') \zeta [(z-z')^2 + \zeta^2]^{-3/2} dz',$$

$$w = -w_0 - \frac{w_0}{2\varkappa^2} \int_{-a}^{a} f(z') f'(z') (z-z') [(z-z')^2 + \zeta^2]^{-3/2} dz'.$$

Reconverting to the initial variables and passing to the limit as $k \to 0$ gives the required velocity field

$$w = -w_0,\quad \sigma = \begin{cases} -w_0 f(z) f'(z) r^{-1}, & |z| < a, \\ 0, & |z| > a. \end{cases}$$

In the reference system moving with the fluid at rest at infinity the vertical velocity component is equal to zero. The fluid moves in such a way that it "separates" in front of the body, flowing in horizontal planes, and "closes up" behind it.

A similar flow structure is also observed in the plane problem. Here too it is possible to employ the slender body approximation. For the vertical motion of a symmetrical body $x = \pm f(z)(|z| < a)$ we obtain the velocity field

$$w = -w_0,\quad u = \begin{cases} -w_0 \operatorname{sign}(x) f'(z), & |z| < a, \\ 0, & |z| > a. \end{cases}$$

it is easy to see that in the plane case the limiting solutions do not decay at infinity. At the same time, in the solutions with finite α there are no perturbations at infinity. This means that in the limit as $\alpha \to 0$ the conditions (2.4), (2.6) break down.

In general, when considering horizontal motion it is not possible to pass to the limit as $k \to 0$. However, it is possible to make a passage to the limit in the equations of motion and then solve the limiting equations obtained. The fact that the solutions of the limiting equations coincide with the limits of the solutions of problem (2.8) can be demonstrated with reference to concrete examples (e.g., for an ellipsoid or an elliptic cylinder). Let us consider the three-dimensional problem. Setting k = 0 in (2.5), (2.6), we find

$$\varphi_{xx} + \varphi_{yy} = 0, \nabla\varphi \to 0 \text{ as } |\mathbf{r}| \to \infty, \quad \varphi_x f_x + \varphi_y f_y = \mathbf{u}_0 \nabla f \text{ when } f = 0. \tag{4.1}$$

The problem can be divided into a set of plane problems in horizontal planes (z = const). Considering that $w_1 + w_0 = -\varkappa^{-2}\varphi_z$, from (4.1) we may conclude that in the horizontal motion of a three-dimensional body the vertical velocity component is equal to zero, the fluid moves only in the planes z = const, and in each of these there is a plane potential flow regime. The same result was previously mentioned in [12].

In the case of the horizontal motion of a plane body it is convenient to make the passage to the limit $k \to 0$ in problem (2.8). At this limit the deformed surface (2.9) is a vertical plate, i.e., (2.8) is the problem of the potential motion of a plate. Since its solution is known, in order to obtain the required velocity field it is sufficient to make a reverse change of variables. In the reference system moving with the body this takes the form:

$$w = 0, \ u = \begin{cases} 0, & |z| < a, \\ -u_0 \dfrac{|z|}{\sqrt{z^2 - a^2}}, & |z| > a \end{cases}$$

for any body whose surface can be given by the equation f(x, z) = 0, |z| < a. It should be noted that this velocity field is determined only by the vertical dimension of the body, and does not depend on its shape. It is also clear that in the limit as $k \to 0$ the conditions at infinity break down, as in the vertical motion of a plane body.

Thus, the limiting flows have a characteristic layered structure. In these flows the vertical velocity component is equal to zero and the motion takes place only in horizontal planes.

5. Resistance to Uniform Motion. In order to calculate the limiting ($k \to 0$) value of the resistance (3.2) it is necessary to know the behavior of the tensor M_{in} near the point k = 0. For a triaxial ellipsoid from the expressions given in [10] we can obtain $M_{11} \sim k^2$, $M_{22} \sim k^2$, $M_{33} \sim k^4 \ln k$, from which for the components of the force (3.4) there follows $F_1 \sim k$, $F_2 \sim k$, $F_3 \sim k \ln k$. Hence, in the limit as $k \to 0$ the resistance to the motion of the ellipsoid in any direction is equal to zero.

For the plane problem the result is fundamentally different: as $k \to 0$ the resistance tends to a finite limit. Thus, for an elliptic cylinder the force (3.7) gives the limit

$$(F_1, F_3) = -\pi\varepsilon N(c^2 u_0, a^2 w_0). \tag{5.1}$$

The qualitative difference in the resistance to the uniform motion of three-dimensional and two-dimensional bodies becomes understandable when the energy relations are considered. The work done by the body $\partial E/\partial t = -F_n u_{*n}(t)$. In accordance with (3.5), this expression can be rewritten in the form $2\alpha E = -F_n u_{*n}$. From this it follows that the resistance is equal to zero if as $\alpha \to 0$ the energy $E(\alpha)$ increases more slowly than α^{-1}, as in the three-dimensional case. For the motion of a plane body $E(\alpha) \sim \alpha^{-1}$ when $\alpha \ll 1$, which corresponds to finite values of the resistance.

Thus, for the uniform motion of a three-dimensional body in an arbitrary direction there is no resistance, i.e., a result analogous to the D'Alembert paradox is obtained. For the uniform motion of a plane body the resistance is finite.

6. Relation to the Problem of Impulsive Motion. The expression (5.1) for the horizontal component of the force F_1 coincides with the value obtained in [3] by passing to the limit as $t \to \infty$ in the problem of the instantaneous setting in motion of a body initially at rest. This coinciding of the limits is not a question of chance. We will show that it is always the case, both for the force and for the velocity field.

The problem of impulsive motion can be formulated as follows. It is required to solve the equations

$$u_t = -p_x, \ v_t = -p_y, \ w_t = -p_z - \rho g, \quad \rho_t + \rho_0'(w + w_0) = 0, \ u_x + v_y + w_z = 0 \tag{6.1}$$

with the boundary conditions

$$\mathbf{u} \to -\mathbf{u}_0 \text{ as } |\mathbf{r}| \to \infty, \ \mathbf{u}\nabla f = 0 \text{ when } f = 0. \tag{6.2}$$

Since the body is instantaneously accelerated, at the initial instant (t = 0) the motion must be potential and must satisfy the boundary conditions (6.2), i.e., $\mathbf{u}(t=0)=\nabla\varphi_0$. It is convenient to take the velocity potential φ_0 in the form $\varphi_0=-\mathbf{u}_0\mathbf{r}+\chi$. Applying to (6.1) a Laplace transformation with respect to time, we obtain

$$s\bar{u}+u_0=\bar{\Phi}_x,\, s\bar{v}+v_0=-\bar{\Phi}_y,$$
$$s\bar{w}+w_0=q^{-2}\bar{\Phi}_z,\ \bar{u}_x+\bar{v}_y+\bar{w}_z=0. \tag{6.3}$$

Here, $\bar{f}(s)=\int\limits_0^\infty \exp(-st)f(t)\,dt$; $\bar{\Phi}\equiv-\bar{p}+\chi$; $q^2\equiv 1+N^2/s^2$; N is the buoyancy frequency; a bar denotes the transform of the corresponding function. The boundary conditions are as follows: $\bar{\mathbf{u}}\nabla f=0$ when f = 0, and $s\bar{\mathbf{u}}\to-\mathbf{u}_0$ as $|\mathbf{r}|\to\infty$. In terms of the function $\bar{\Phi}$ the problem takes the form:

$$\bar{\Phi}_{xx}+\bar{\Phi}_{yy}+q^{-2}\bar{\Phi}_{zz}=0,$$
$$\bar{\Phi}_x f_x+\bar{\Phi}_y f_y+q^{-2}\bar{\Phi}_z f_z=\mathbf{u}_0\nabla f \text{ when } f=0,\quad \nabla\bar{\Phi}\to 0 \text{ as } |\mathbf{r}|\to\infty. \tag{6.4}$$

Solving the problem (6.4), we find the function $\bar{\Phi}$. Then Eqs. (6.3) give $\bar{u}$, $\bar{v}$, $\bar{w}$.

In order to compare problems (2.5), (2.6), and (6.4) it is convenient to consider the case of real s. We pass to the limit as s → 0 by (6.3). By virtue of the well-known [13] property of the Laplace transformation

$$\lim_{s\to 0} s\bar{f}(s)=\lim_{t\to\infty} f(t),\ |\arg s|<\pi/2-\delta$$

we obtain

$$u_0+\lim_{t\to\infty} u=\lim_{s\to 0}\bar{\Phi}_x,$$
$$v_0+\lim_{t\to\infty} v=\lim_{s\to 0}\bar{\Phi}_y,\ w_0+\lim_{t\to\infty} w=\lim_{s\to 0} q^{-2}\bar{\Phi}_z. \tag{6.5}$$

The function $\bar{\Phi}$ in (6.5) is the solution of problem (6.4), which formally coincides with the problem (2.5), (2.6) previously formulated in studying exponential motions. When relations (2.3) and (6.5) are taken into account, this means that the velocity field obtained in the problem of impulsive motion in the limit as t → ∞ coincides with the velocity field obtained in solving (2.5), (2.6) in the limit as α → 0, which corresponds to uniform motion.

The asymptotic expression for the resistance as t → ∞ is similarly calculated:

$$F_i=-\lim_{t\to\infty}\int\limits_{\partial\tau} p\,dS_i=\lim_{s\to 0} s\int\limits_{\partial\tau}\bar{\Phi}\,dS_i. \tag{6.6}$$

It is easy to see that correct to the notation (6.6) coincides with the corresponding expression (3.2) taken in the limit as α → 0. It has thus been shown that the limiting (α → 0) solutions considered coincide with the solutions of the problem of the impulsive motion of a body in a stratified fluid taken in the limit as t → ∞.

We conclude with the following remarks.

1. The solutions constructed are characterized by the parameters ε and k. If, following [6], we sum the various combinations of ε and k, then we can solve the problem for the laws of motion $\mathbf{u}_*(t)$ represented in the integral form:

$$\mathbf{u}_*(t)=\mathbf{u}_0\int\limits_0^{k_0}\varepsilon(k)\exp(kNt)\,dk,\ 0<k_0\leqslant\infty,$$

where ε(k) is an arbitrary function that ensures the convergence and smallness of the integral. In this case the formulas for the resistance are obtained by integrating relations (3.2) with respect to k.

2. The case of oscillatory motions $\mathbf{u}_*(t)=\varepsilon\mathbf{u}_0\cos(\alpha t)$ with frequency α greater than the buoyancy frequency N can be considered in exactly the same way as the case of exponential motions (1.5).

3. The approach proposed is also applicable to the motion of a body in straight channels, both horizontal and vertical.

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EFFECT OF AN INFLECTION IN THE PROFILE OF MEAN VELOCITY ON THE RESONANCE INTERACTION OF PERTURBATIONS IN A BOUNDARY LAYER

M. B. Zel'man and B. V. Smorodskii

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The character of the laminar-turbulent transition (LTT) in shear flows depends to a considerable extent on the distribution of the vorticity of the average motion. According to the linear theory of stability, the appearance of extrema in such distributions (points of inflection in the velocity profile) leads to expansion of the spectrum and an increase in the increments of unstable pulsations that are already taking place (see [1, 2]). Both the time of formation of the nonlinear regime and the character of its occurrence are variable.

The appearance of inflections may be due either to external flow conditions or to the nonlinear self-perturbation of "primary" waves in the flow. Examples of the effect of such mean flow singularities on the interaction of wave perturbations were examined in [3] for free shear layers and in [4] for pre-separation boundary layers. However, the laws governing the evolution of interacting waves under these conditions have yet to be definitively established.

The goal of the present investigation is to explore features of the effect of the characteristics of inflected profiles on resonance wave interactions in boundary layers. The results that are obtained are used to interpret the mechanism responsible for preventing the occurrence of a subharmonic S-type transition with an increase in the level of the initial perturbations.

We choose a flow with the profile $U_G(y)$ [5] as the initial flow for studying the evolution of resonance perturbations. This flow models the motion of intensive eddies in a boundary layer:

$$U_G(y) = U_\pm + \varkappa(\mathrm{th}\,(y - y_r)/\delta \mp 1),\ y \gtrless y_r.$$

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